

*Application of Normal Mode Expansion to AE Waves in Finite Plates*¹

M. R. Gorman² and W. H. Prosser³

INTRODUCTION

Breckenridge et al. (1975), Hsu (1985) and Pao (1978) adapted approaches from seismology to calculate the response at the surface of an infinite half-space and an infinite plate. These approaches have found use in calibrating acoustic emission (AE) transducers. However, it is difficult to extend this theoretical approach to AE testing of practical structures. Weaver and Pao (1982) considered a normal mode solution to the Lamb equations. Hutchinson (1983) pointed out the potential relevance of Mindlin's plate theory (1951) to AE. Pao (1982) reviewed Medick's (1961) classical plate theory for a point source, but rejected it as useful for AE and no one seems to have investigated its relevance to AE any further.

Herein, a normal mode solution to the classical plate bending equation was investigated for its applicability to AE. The same source-time function chosen by Weaver and Pao is considered. However, arbitrary source and receiver positions are chosen relative to the boundaries of the plate. This is another advantage of the plate theory treatment in addition to its simplicity. The source does not have to be at the center of the plate as in the axisymmetric treatment. The plate is allowed to remain finite and reflections are predicted. The importance of this theory to AE is that it can handle finite plates, realistic boundary conditions, and can be extended to composite materials.

¹ Work supported by NASA Langley Research Center

² Dept. of Physics
University of Denver
Denver, CO 80208-0177

³ Nondestructive Measurement Science Branch
NASA LaRC, Hampton, VA 23681-0001

THEORETICAL

The normal mode solution procedure has been widely applied to a number of elasticity problems including strings, beams, membranes, and plates. This solution technique is discussed in detail by Graff (1991). For a thin, homogeneous, isotropic plate, the governing equation of motion from classical plate theory is

$$D\nabla^4 w(x,y,t) + \rho h \partial^2 w(x,y,t) / \partial t^2 = f(x,y,t) \quad (1)$$

where the x and y axes lie in the plane of the plate and the z axis is perpendicular to the plane of the plate. Time is represented by t , $\nabla^4 = (\partial^2 / \partial x^2 + \partial^2 / \partial y^2)^2$ and $f(x,y,t)$ is the force (N/m²) applied in the z direction. $w(x,y,t)$ is the transverse displacement, ρ the density, and h the plate thickness. D is the bending stiffness given by

$$D = Eh^3 / 12(1-\nu^2) \quad (2)$$

where E is Young's modulus and ν is Poisson's ratio.

A step forcing function at $x=\xi$, $y=\zeta$, given by

$$f(x,y,t) = P\delta(x-\xi)\delta(y-\zeta)H(t) \quad (3)$$

where P is the amplitude of the step is used and the plate is assumed to be simply supported at its boundaries, $x=0$, $x=l$, $y=0$ and $y=w$. The resulting normal mode solution is given by

$$w(x,y,t) = \frac{4P}{\rho h l w} \sum_{n,m} \frac{\sin \alpha_n x \sin \gamma_m y \sin \alpha_n \xi \sin \gamma_m \zeta (1 - \cos \omega_{nm} t)}{\omega_{nm}^2} \quad (4)$$

where $\alpha_n = n\pi/l$ and $\gamma_m = m\pi/w$ for $n, m = 1, 2, \dots$,

$$\omega_{nm} = \pi^2 \left[\frac{n^2}{l^2} + \frac{m^2}{w^2} \right] a \quad (5)$$

and $a^2 = D/\rho h$.

This result is to be compared with Medick's (1961) result for an infinite plate subject to a step function in time at the origin

$$w(r,t) = \frac{P}{4\pi(\rho h D)^{1/2}} t H(4ar^2/t) \quad (6)$$

where

$$H(x) = \pi/2 - Si(x) + xCi(x) - \sin(x) \quad (7)$$

with $Si(x)$ and $Ci(x)$ being the sine and cosine integrals, respectively. Equations (4) and (6) were programmed and evaluated for several source to receiver distances for an aluminum plate and compared with experimental data.

EXPERIMENTAL

A pencil lead break provided a source which approximates a vertical step function, but the magnitude of the force was not measured. The aluminum plate had dimensions $l=0.381$ m., $w=0.508$ m., and $h=0.003175$ m. A 1.27 cm. diameter 3.5 MHz ultrasonic transducer (Panametrics) was used as the receiver because of its wideband behavior in the frequency range of interest (20-500 KHz). Papadakis (1980) pointed out that these transducers should provide flat frequency response from near zero frequency to just below their resonance frequency and thus make high fidelity AE sensors. The large diameter of this sensor is not a concern in these measurements because of the long wavelengths of the flexural mode signals being measured. This sensor has been shown to be displacement sensitive in previous research by Prosser (1991). Although the frequency response of this sensor is much flatter and broader than conventional resonant AE sensors, its response does drop off at low frequencies. The frequency response of the transducer was determined and applied to the theoretical calculations. Because the frequency of each normal mode summed in the theoretical calculation is known, it was easier to apply the transducer filter response to the theoretical calculations than to the measured data.

RESULTS AND DISCUSSION

Figure 1 shows a comparison between our normal mode solution and Medick's integral transform solution for the same source to receiver distance on the aluminum plate. This distance and the length of time shown in the plot in this figure were chosen such that there are no reflections from the plate boundaries included in the waveforms. Reflection signals are predicted in the normal

mode solution but neglected by the Medick (1961) infinite plate solution. Also, since this is a comparison of theory with theory, the transducer response filtering was not applied. It can be seen that the agreement is quite good for $n=m=100$. From equation (5), the highest frequency term in the normal mode solution is 834 KHz which is well above those seen in the experimental data. The 10,000 terms took less than five minutes to compute on a 68030 based microcomputer for 512 points.

Figures 2 and 3 show the measured waveforms and the theoretical waveforms at distances of 0.1016 m. and 0.1778 m. from the source. According to observations by Medick (1961), this theory should agree with experiment when the wavelength is greater than 16 times the thickness of the plate. This corresponds to a time greater than

$$\tau_c = 2 \left(\frac{x}{c} \right) . \quad (8)$$

where c is the characteristic velocity $(E/\rho)^{1/2}$. This implies that theory and experiment should begin to agree at the point indicated by the arrow in the figures. It can be seen that there is good agreement beyond this initial critical time. Note that the extensional mode can be seen in each of the experimental waveforms. Due to the higher wavespeeds of the extensional mode, it begins to separate from the flexural mode as the source to receiver distance increases. Figure 4 shows the response for longer times. It can be seen that there are reflections present in both theory and experiment which would not be predicted by theory for an infinite plate. It is interesting to note, as Medick (1961) does, that the quantitative predictions of the classical theory retain their accuracy even for points only several plate thicknesses removed from the loading area as illustrated for the response of the plate at a distance of 0.1016 m from the source.

ACKNOWLEDGEMENT

We are grateful to NASA for supporting this work. E. I. Madaras, Langley Research Center, was the program monitor.

REFERENCES

Breckenridge, F. R., Tschiegg, C. E., and Greenspan, M., 1975, "Acoustic Emission: Some Applications of Lamb's Problem," *J.*

Acoust. Soc. Am., Vol. 57(3), pp. 626-631.

Graff, K. F., 1991, *Wave Motion in Elastic Solids*, Dover Publications, New York.

Hsu, N. N., 1985, "Dynamic Green's Functions of an Infinite Plate - A Computer Program," *NBSIR 85-3234*.

Hutchinson, R., 1983, "Axisymmetric Elastic Waves Excited by a Point Source in a Plate," *J. Applied Mechanics*, Vol. 50, pp. 699-700.

Medick, M. A., 1961, "On Classical Plate Theory and Wave Propagation," *J. Applied Mechanics*, Vol. 28, pp. 223-228.

Mindlin, R. D., 1951, "Influence of Rotary Inertia and Shear on Flexural Motions of Isotropic, Elastic Plates," *J. Applied Mechanics*, Vol. 18, pp. 31-38.

Pao, Y. H., 1978, "Theory of Acoustic Emission," *Elastic Waves and Non-Destructive Testing of Materials, AMD - Vol. 29*, pp. 107-128.

Pao, Y.H., 1982 "Transient AE Waves in Elastic Plates," *6'th International AE Symposium*, Japan, pp. 181-197.

Papadakis, E. P., 1980, "Broadband Flaw Detection Transducers: Application to Acoustic Emission Pulse Shape and Spectrum Recording Based on Pulse Echo Response Spectrum Corrected for Beam Spreading," *Acoustica*, Vol. 46, pp. 293-298.

Prosser, W. H., 1991, "The Propagation Characteristics of the Plate Modes of Acoustic Emission Waves in Thin Aluminum Plates and Thin Graphite/Epoxy Composite Plates and Tubes," NASA Technical Memorandum 104187.

Weaver, R. L., and Pao, Y. H., 1982, "Axisymmetric Elastic Waves Excited by a Point Source in a Plate," *J. Applied Mechanics*, Vol. 49, pp. 821-836.

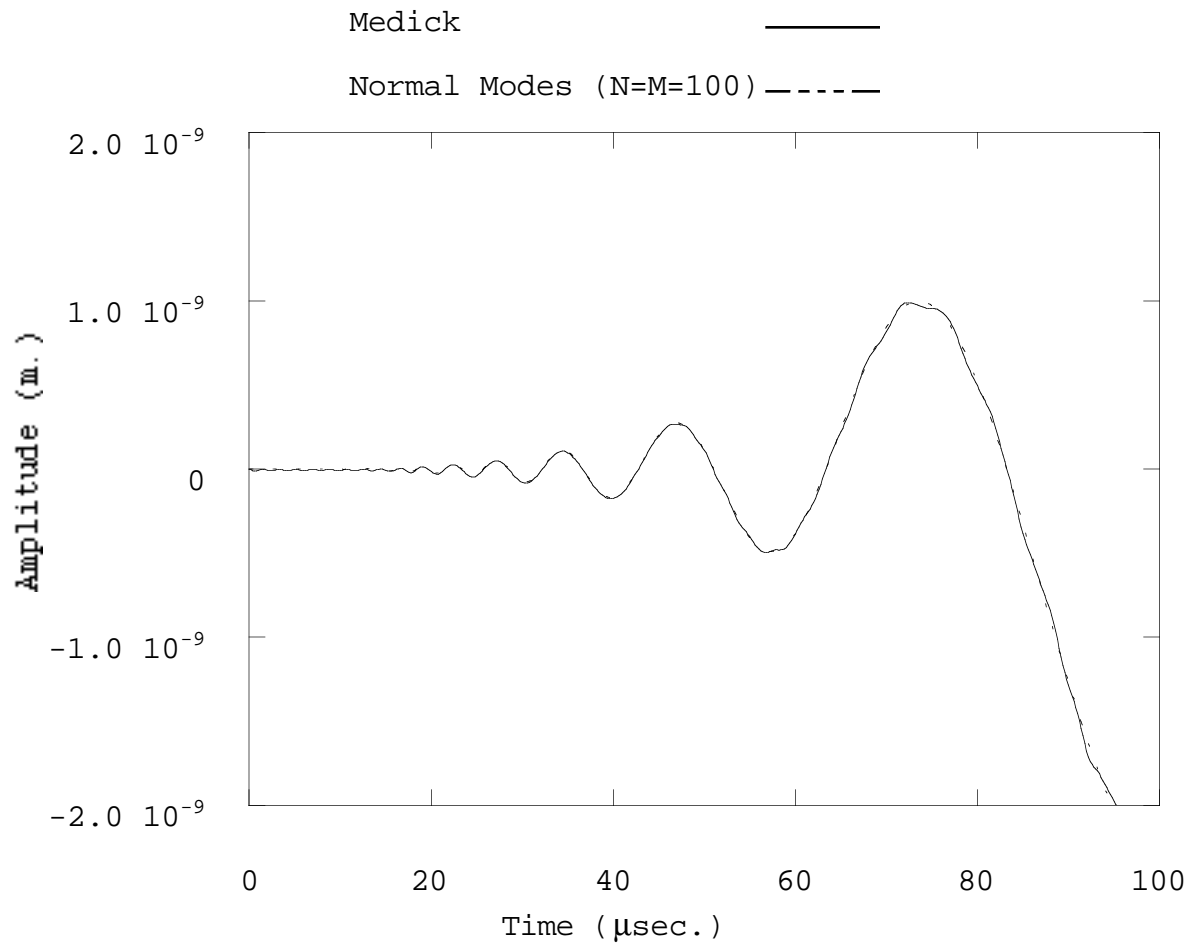


Figure 1. Plate response to a step function for a source to receiver distance of 0.127 m.

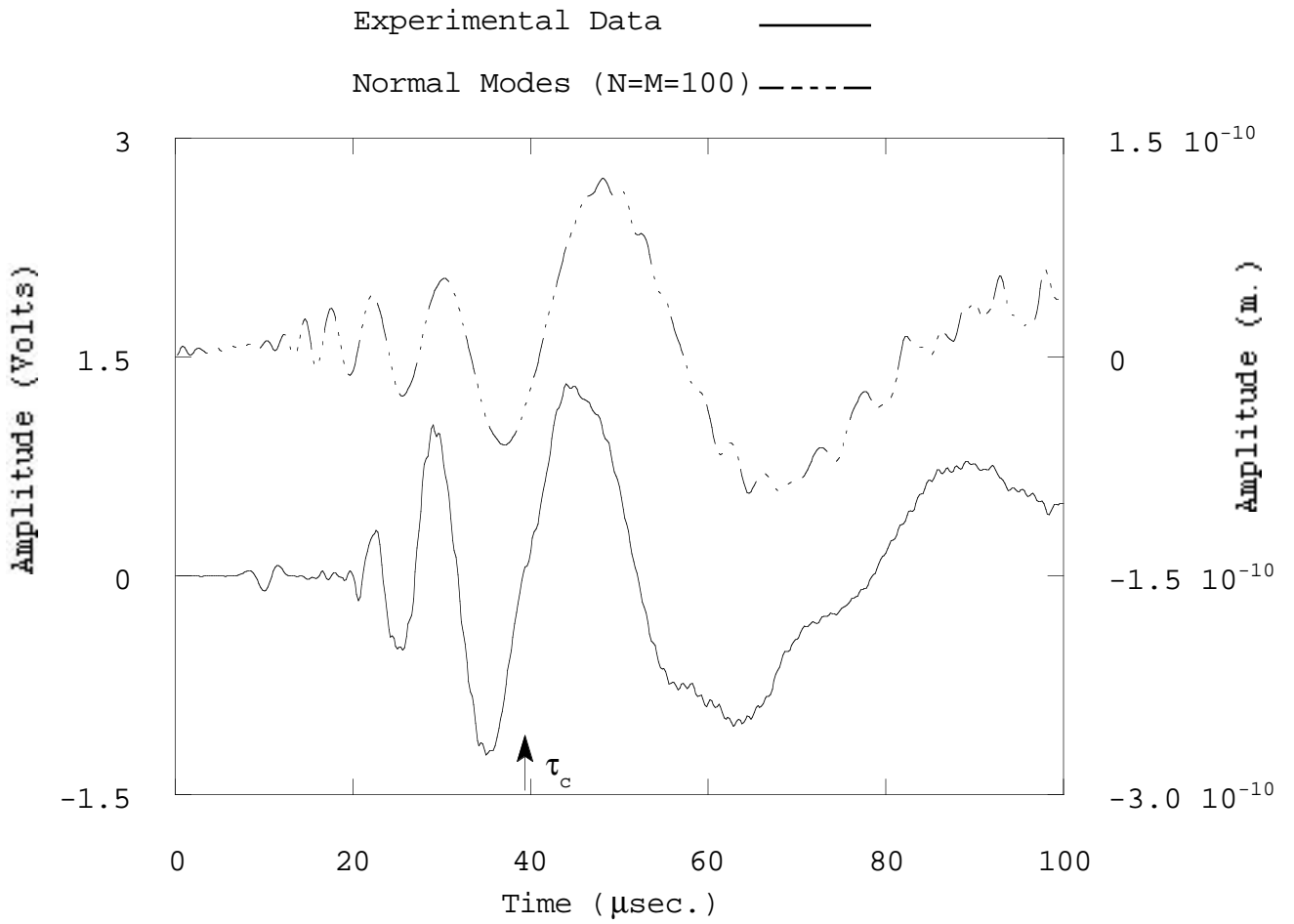


Figure 2. Lead break at 0.1016 m from receiver.

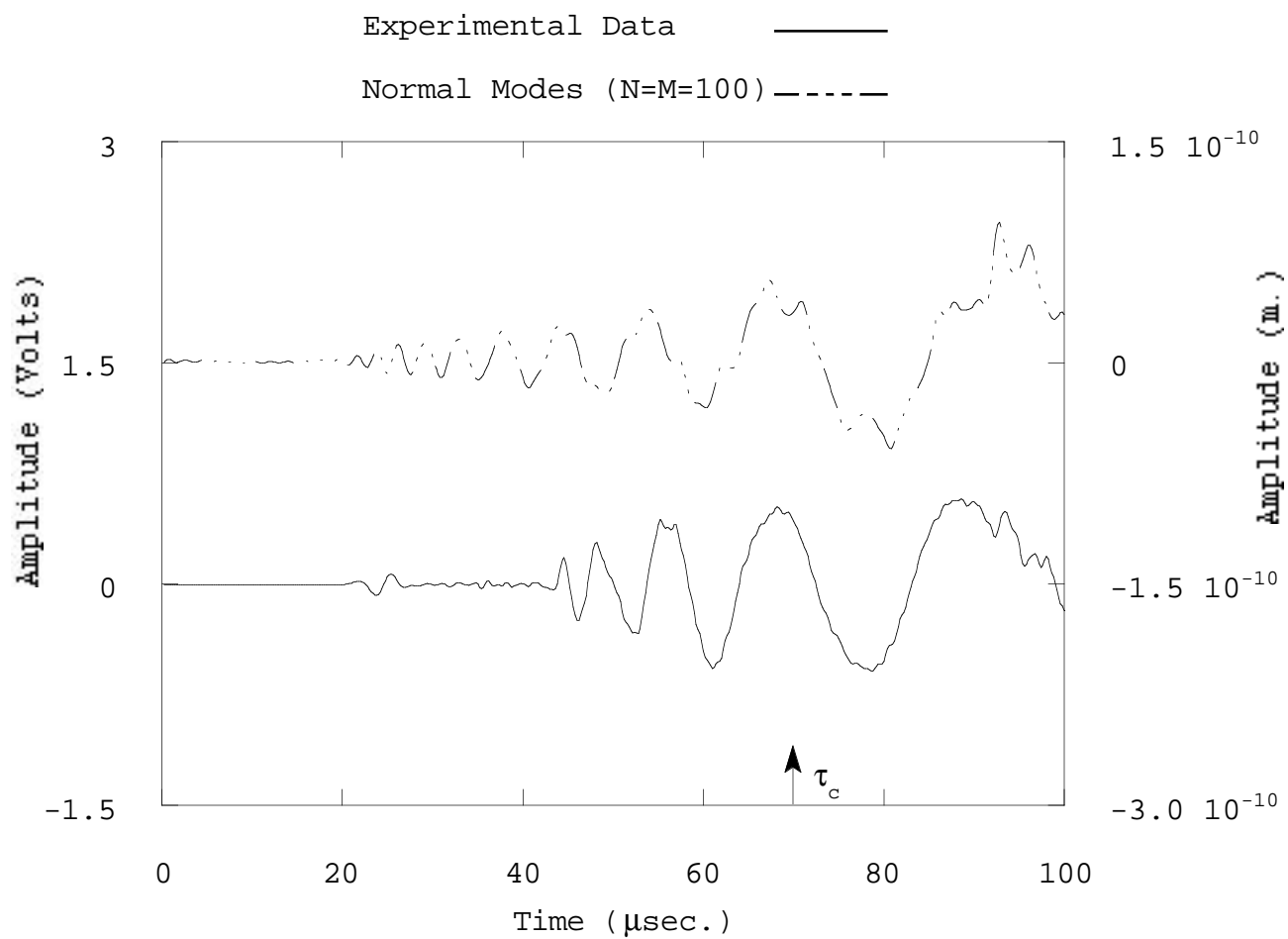


Figure 3. Lead break at 0.1778 m from receiver.

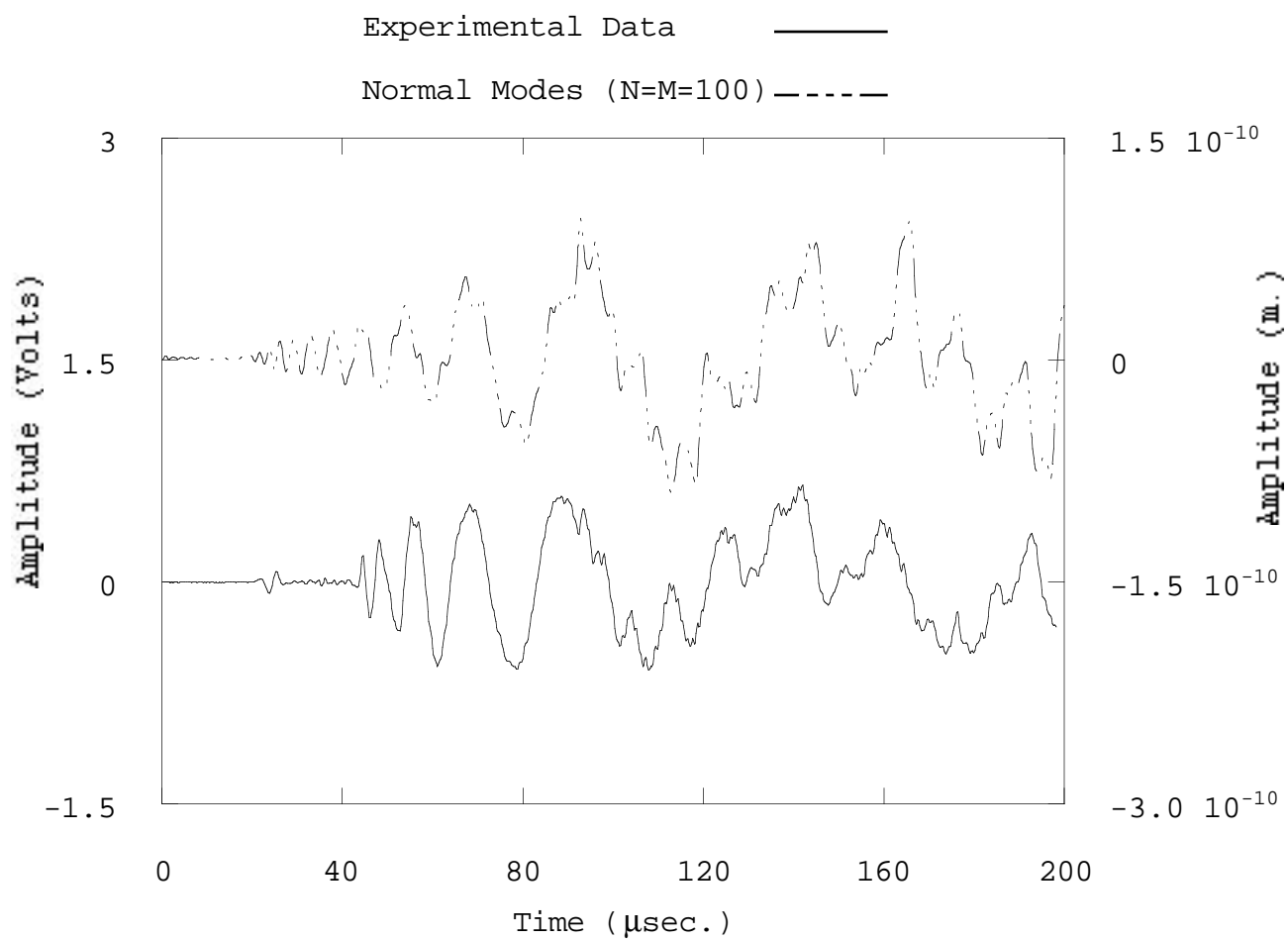


Figure 4. Theory and experiment compared at longer time shows reflections.